文章编号　1004-924X(2001)03-0203-09

Multiresolution Surface Construction

Guo Yun[1], Yasushi Yamaguchi[1], Zhong Xian-xin[2]

(1. *Department of Graphics and Computer Sciences, The University of Tokyo, Japan;*
2. *Department of Optical Mechanics, Chongqing University, China*)

Abstract: Unstructured and highly detailed meshes need to be effectively represented in many applications. Accompanied with wavelets, hierarchical multiresolution meshes can be in charge of this effective representation. To perform this multiresolution representation with wavelets, an available function space needs to be constructed and defined over the general triangulated surface meshes. To employ an effective method to accomplish the construction of a continuous parameterization of arbitrary meshes over a simple domain is a key problem. The paper proposes a fast parameterization method based on geodesic polar map that can be effectively served in multiresolution surface construction. Compared with previous methods, the construction process given in the paper can save a lot of computation time and maintain fine visual result.

Key words: multiresolution surface representation; parameterization; geodesic polar map; wavelets
CR Categories and Subject Descriptors: I. 3. 5[Computer Graphics]: Computational Geometry and Object Modeling. - Surfaces and Object Representations.

1　Introduction

There are many representations for surfaces used in computer graphics such as parametric surfaces and constructive solid geometry etc. Triangle meshes, supported by the graphics hardware and obtained conveniently from laser scanning system, are capable of describing surfaces of arbitrary shape and topology. With the advent of laser scanning system, arbitrary objects can be made up of dense triangular meshes, which possibly consist of large amount of triangles[17]. To apply these triangular objects to various fields, such as CAD, reverse engineering, arts etc., one need to find more effective technologies to process these objects. The first problem to be considered is to reconstruct the objects to be suitable for application in current computer capabilities. Multiresolution representation has been used as an empirical technology to produce applicable surfaces. Classical multiresolution technology based on wavelets when applied to geometric problems needs to seek a functional space over the original triangulated 2-manifold mesh[1,6,10]. To implement this transformation to function space, an important element is concerned with parameterization, which sets up the complexity domains to support triangulated surface parameterization. Founded on this fact, two methods, harmonic map[1,6] and its variant PL harmonic embedding[3] have been employed in the triangulated surface multiresolution analysis.

In this paper, we draw on the parameterization method[14] based on geodesic polar map, and apply it to our multiresolution surface construction practice. From the differential character of geodesic polar map, we named this parameterization shape-preserving parameterization. The computation time and sampling results derived from the harmonic map method and shape-preserved method show that the latter method is more effective when used in multiresolution surface construction for the triangulated surface. Besides, we simultaneously combine smaller patches (compared with the strategy used in[6], figure 3) to do parameterization, which also contributed to less computation time.

收稿日期: 2001-02-14; 修订日期: 2001-04-26

2 Related Work

The classical theory for multiresolution signal decomposition with the help of wavelet representation indicates that a signal can be approximated by a given resolution. A family of wavelet basis functions can withdraw the difference of information between two adjacent given resolutions[5]. This enables us to have a hierarchical representation for the original signal with gradually increasing resolution such as 2^j ($j \in Z$). These wavelets have been introduced as some functions with the characters of translations and dilations in many areas including image compression, numerical analysis and physical simulation. Michael Lounsbery and his coworkers[7] have developed a class of wavelets, based on the subdivision surface, applied to functions defined on compact surfaces of arbitrary topological type.

Eck. etc.[6] firstly provided an appealing framework for the multiresolution geometric surface construction which originated from the view of a mathematical formalism of wavelet multiresolution analysis.

The key component of[6] is to set up the smooth parameterization of triangulated surfaces over simply complex base domains, which include some Delaunary-like triangular regions generated first by constructing a Voronoi graph. In order to reach the goal, a series of approximate harmonic maps are employed to accomplish parameterization. In[6], harmonic maps have been repeatedly used in three main stages. These stages can be generalized as the following: from Voronoi tiling to Delaunary triangular, combining some subpatch of Voronoi tiles to straight the edge of Delaunary triangular regions and the initial domains construction. The initial domains and the parameterization over it build the function description of the original triangulated surface. This description makes it practicable to hierarchically represent the surfaces by subdivision schemes like Loop[8]. The sampled vertices with subdivision connectivity from the original triangulated surface can be used to construct the wavelet representation. The original triangulated surface can be further reconstructed from a wavelet representation. The multiresolution analysis using wavelets can go with this remesh.

From the point of view of graph with the consideration of global geometric features, P. Gioia[1] proposed another measure to implement the multiresolution analysis, which reduced the number of wavelet coefficients for equal detail representation. Resembling as[6], some approximate harmonic maps have been used to build a parameterization of original surface meshes over triangular regions with good geometric approximation.

Lee[10] provided an algorithm to consider the local sharp features to construct a hierarchical parameterization over the base domain space. The parameterization is implemented by using the PL harmonic embedding operation.

The above-mentioned schemes heavily made use of harmonic maps to implement the parameterization. Different from those work, we present an alternative approach to the parameterization in multiresolution surface construction using the shape-preserving method. It results in less computing time and better sampling effect than those with harmonic map method at the time when triangulated surface reconstruction is conducted. In next section we will discuss the two main parameterization methods and make some comparisons. In section 4 we will outline the results of our multiresolution surface construction practice. In the final section, conclusions are discussed and further work are presented.

3 Parameterization

3.1 Harmonic Maps

In triangulated surface parameterization, harmonic map has already been widely used in different areas such as surface match[16] and graphics morphing[15], especially in the region of multiresolution surface construction. Its elementary thought is as below:

We suppose $f: \Omega \subset N \to M$ is a smooth map between Riemannian manifolds (N, h) and (M, g) with metrics h and g, respectively. This is a parameterization of surface $f(\Omega) \subset M$ over a two dimensional submanifold $\Omega \subset N$. The Dirichlet energy of the map f is defined as

$$E_d(f) = 1/2\int_{\Omega} | \nabla f |_g^2$$

where $| \nabla f |_g^2 =$ trace g (∂ f, ∂ f, ∂ f, ,,,). g is served as metric. The critical points of this energy function are called harmonic maps. This is a general definition of harmonic maps. Now, we consider the special case. The two manifolds are in the 3D Euclide space. The M is a surface of disc topology and N is a planar region. The harmonic map theory leads to a unique solution when we give a homeomorphism φ between the two boundaries of M and N for this map[2]. Furthermore if we let Ω be a domain with triangulation Γ_h and ω_h be the set of continuous and piecewise linear functions on Γ_h. Then we can define the discrete Dirichlet energy of any function $f_h \subset \Gamma_h$:

$$E_d(f_h) = 1/4 \sum_{edge\{i,j\}} (\cot \alpha_{ij} + \cot \beta_{ij}) | f_h(x_i) - f_h(x_j) |^2$$

[1, 6] took an approximate approach to the harmonic map. Given firstly a homeomorphism Φ between the boundary of f(Ω) and the boundary of Ω, the map can be imaged as the deformation of a network of springs. Then the boundary of Ω can be chosen according to the concrete application. The energy function can be computed as follows:

$$E_d(f) = 1/2 \sum_{\{i,j\} \in \text{edges}(\Omega)} K_{ij} \| f(i) - f(j) \|^2 \quad (1)$$

$f(i)$ and $f(j)$ are the images of the vertices i and j of $f(\Omega)$ on Ω. The K_{ij}, serving as the spring constants, is related with the two triangles adjacent to the edge $\{i, j\}$. The energy $E_d(f)$ can be regarded as the result from the deformation of triangulated spring meshes. In approximate harmonic maps, the spring coefficient K_{ij} is destined to preserve the ratios of edge length as small as possible in image space. For the two defined vertices of one edge $\{i,j\}$, the K_{ij} is always equal. Minimizing the energy function $E_d(f)$ is actually to solve the symmetric sparse linear system:

$$\Delta E_d(f) = \partial E_d(f)/\partial V_i = 2H_{ii}V_{ii} + 2H_{ib}V_b = 0 \quad (2)$$

where V_i is a vector of images corresponding to all interior vertices in Ω. V_b is a vector of images of boundary vertices of Ω. A general conjugate gradient algorithm is preferable to solve the linear equations. Figure 1(c) displays an approximate harmonic map result.

3.2 Shape-preserving Method based on Geodesic Polar Map

Unlike previous work, we use shape-preserving method based on geodesic polar map in our multiresolution surface construction. This method originated from Michael S. Floater's parameterization[14]. He proposed this fast parameterization method and used it in his surface fitting. The foundation of this approach is a geodesic polar map preserving the radial directional differential similarity. The basic idear behind this parameterization is to set each vertex in image space to be a convex combination of its neighbors. And this combination is unique. The details are as follows:

Firstly, let $S(G, X)$ be disc topology surface triangulation with graph $G(V, E, F)$ and node set $X = \{x_i \in R^3, i = 0, 1, 2, ,,, , n-1, n, n+1, ,,, ,N-1\}$. We set the vertex set $V = \{i; i = 0, 1, 2, ,,, , n-1, n, n+1, ,,, , N-1\}$. The range indexes [0, n−1] are pointed to interior vertices and [n, N−1] boundary vertices index. The parameterization domain corresponding to $S(G, X)$ is a triangulated planar region $R(G, P)$ with node set $P = \{p_i \in R^2, i = 0, 1, 2, ,,, , n-1, n, n+1, ,,, , N-1\}$.

Secondly, we ensure the unique convex combination for each node $i \in [0, n-1]$ to be built. Put the boundary of S onto the boundary of R by a piecewise linear function. For our experiments, the boundary of R is a convex polygon $D \subset R^2$ with r corners. The following is the definition for the convex combination in the image space.

$$p_i = \sum_{j=0}^{N-1} \lambda_{ij} p_j \quad \text{with} \quad i \in [0, n-1] \quad (3)$$

where the λ_{ij} chosen by the geodesic polar map in addition to the constrains:

$$\lambda_{ij} > 0 \quad \{i,j\} \in E \quad \text{otherwise } \lambda_{ij} = 0,$$

$$\sum_{j=0}^{N-1} \lambda_{ij} = 1$$

The proof of this the unique of convex combination can be found in[14].

Thirdly, a geodesic polar map needs to be built for each interior vertex $i \in [0, n-1]$. In differential geometry, the geodesic polar map originates from the exponential map[4]. The exponential map carries radial lines from vertex i in tangent plane of i to geodesics starting at vertex i in S. And the exponential map holds the diffeomorphical character for the neighborhood of i between image and source space.

Let G_i be any one subgraph with vertex i and its neighborhood, then a relative local coordinate system with the origination p_i is set as follows. For the neighborhood p_{ij}($j \in [0, d_i - 1]$, degree d_i is the number of neighborhood), the following conditions are satisfied.

. $\| x_i - x_{ij} \| = \| p_i - p_{ij} \|$.

. These angles between x_{ij} around x_i are scale to the related angles between p_{ij}.

At the end of setting this local map, for each neighborhood of i, a triangle $\Delta p_{ij} p_{ir(j)} p_{ir(j)+1}$ including p_i can be generated. However, p_i is completely inside the triangle or on the boundary of the triangle, the barycentric coordinates should exist for p_i with respect to this triangle.

$$p_i = \sum_{k=0}^{d_i - 1} p_{ij} b_{jk},\ b_{jk} = 0 \quad \text{when} \quad k \neq r(j) \quad \text{and} \quad k \neq r(j) + 1. \tag{4}$$

Finally, the coefficients λ_{ij} can be computed as the following:

$$\lambda_{ij} = 1/d_i \sum_{k=0}^{d_i - 1} b_{jk}$$

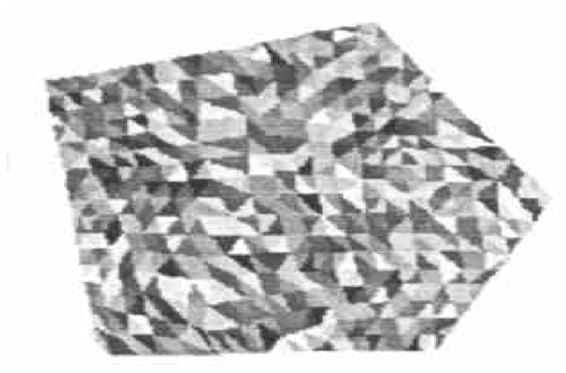
The formula (3) can be written into the following matrix linear equations:

$$BP_T = C \tag{5}$$

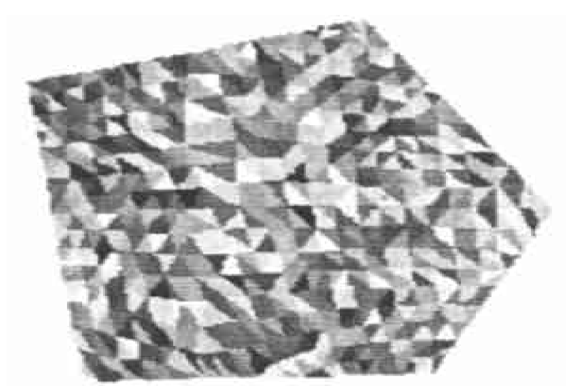
Known from the above discussion, B is a sparse unsymmetrical matrix. Generally speaking, the biconjugate gradient iterative method or generalized minimum resident method can solve the unsymmetrical linear equation questions. In all our parameterization and sampling experiments, the approach named Bi-CGSTAB[9] is adopted to be the solver of the formula (5), which leads to the fast smoothly converging result. Compared with the conjugate gradient algorithm, this solver also holds attractive speed of converging. We have already performed hundreds of the parameterization founded on geodesic polar maps using the Bi-CGSTAB solver. It works well for such linear equations. Fig. 1(b) is the result from shape-preserving parameterization.

3.3 Comparison of the Experiments

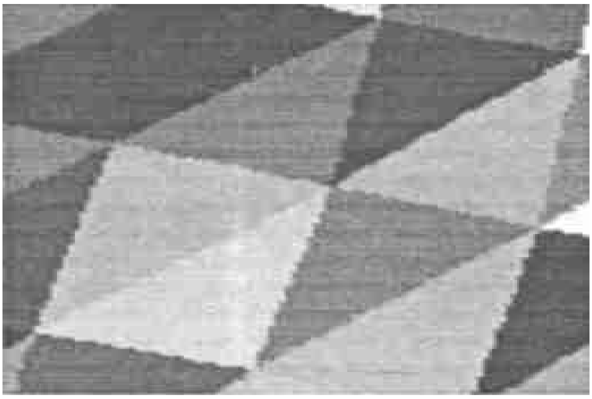
(a) Original surface patch



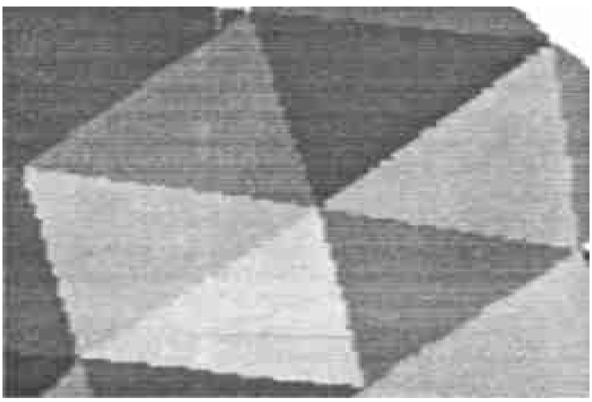
(b) Shape-preserving embedding



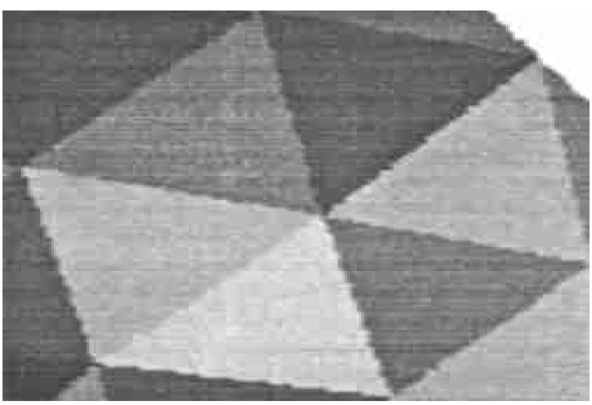
(c) Harmonic embedding



(d) A subpatch of (a)



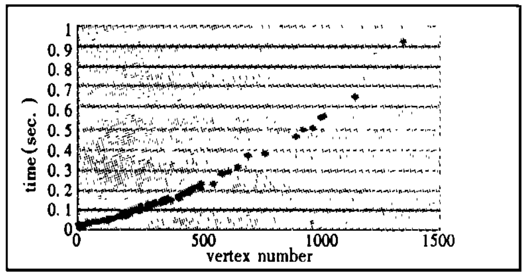
(e) Shape-preserving map for (d)



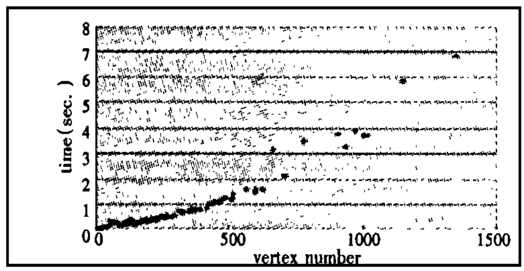
(f) Harmonic map for (d)

Fig. 1 One surface patch cut from Stanford bunny. (d) indicates a vertex and its neighborhood. The shape variation can be observed from (e) and (f) .

In the process of our practice for multiresolution surface construction, the parameterization based on harmonic map and the parameterization based on geodesic polar map have been carried out respectively. As we expected, the latter can preserve the similar local shape in the polygon of R2 to relative surface patch excluding the boundary triangles. Figure 1 (d)-(f) show us the tiny shape differences.



(a) Shape-preserving method



(b) Harmonic map

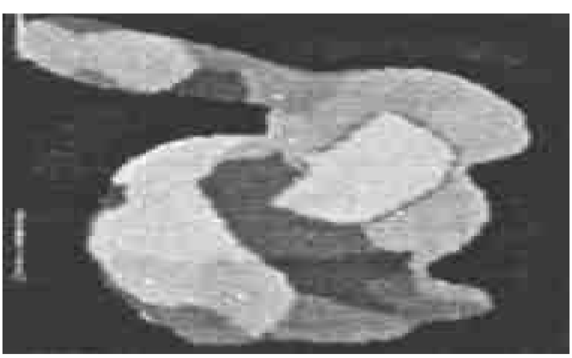
Fig. 2 Computation time for different parameterizations.

We carry out the partitioning work through Voronoi graph for Stanford bunny (see section 4) . Figure 2 shows the computing time comparison, which has been finished according to the parameterization of 92 Voronoi surface patches over the domains of R^2. The start point of this computation time is from fixing the boundary vertex for each patch. Studying the trends from these charts, it is not difficult to find the shape-preserving method with the faster convergence than the scheme based on the harmonic maps. Also, we can feel, for figure 2 (a), the computation time increment is not so much with the increasing of vertex amount. Surface curves sampled from the parameterization of shape-preserving method can be observed from Fig. 3 (c) . By the way, these experiments we have done are under the platform of Windows NT 4.0 and Visual C++ 5.0.

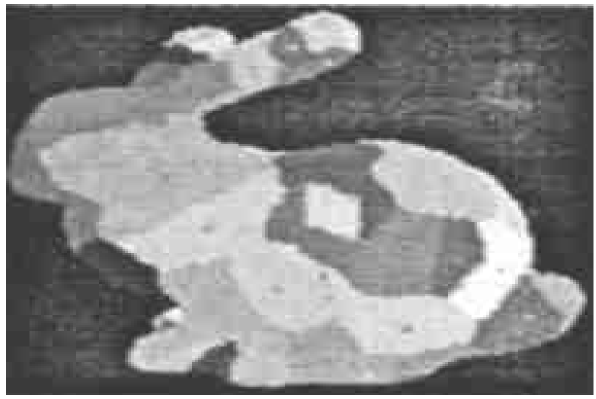
4 Multiresolution Surface Construction

From the signal analysis, we know a signal can be considered in the frequency domain. Therefore, can consider the surfaces as some kind of function about the distributor of a geometric measure such as curvature. Sampling theory tells us that a better-reconstructed signal should be made up of much different frequency component of itself. The surface reconstruction will obey the principle as soon as possible.

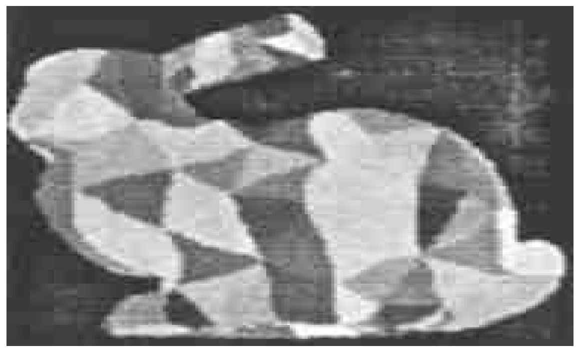
Our main frameworks of multiresolution surface construction come from[6]. At first, a partition was carried out over the densely triangulated surface. Growing a Voronoi diagram using a multi-seed Dijkstra's algorithm can generate the patches. This diagram is then triangulated by the dual of the Voronoi graph and Delaunary-like triangular. Here, in order to straighten the curve edge of Delaunary-like triangular, we combined two patches smaller than[6] (Figure 3 (a), (c)) as a combined domain to perform a parameterization over it. This combined domain apparently encloses fewer vertices so as to reduce the dimension of sparse matrix to be solved (Figure 3 (b), (e)). To reach the approved goal, we strictly limited the conditions of producing the Voronoi diagram. We let each Voronoi site closer to the geometric center corresponding to Voronoi patch. These limitations enable us to produce a good look Delaunary triangular surface patches (Figure 3 (c)). The results are shown in figure 3.



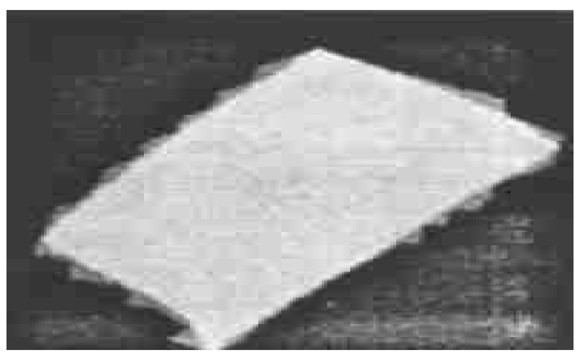
(a) Combined region



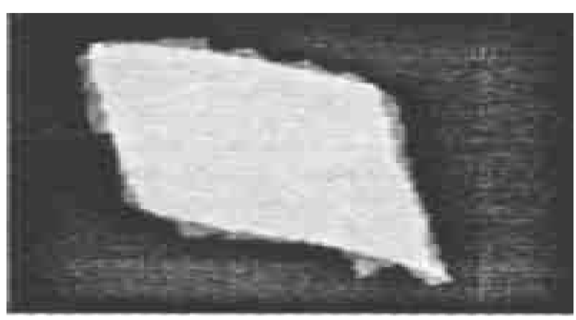
(b) Combined smaller region



(c) Delaunary triangular



(d) Combined patch

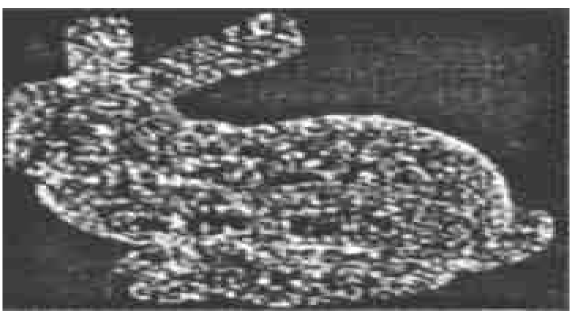


(e) Combined smaller patch

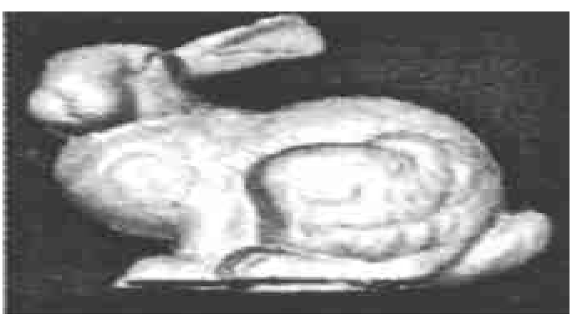
Fig. 3 Different combined domains for (a) and (b). Resulting Delaunary triangular surface patches (c) come from (b) using the shape-preserving parameterization

Next is to complete a parameterization of original surface over the Delaunary-like triangular regions. In our experiments, we used the method based on geodesic polar map narrated as before to implement the parameterization. In figure 3 (c), we set up the continuous parameterization over the domains of 252 Delaunary triangular planar regions.

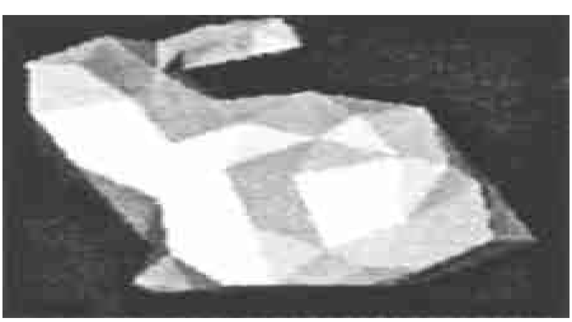
Finally, as the original triangulated surface coming from the real object in the form of uniform-sized triangle meshes, the uniform triangular sampling out of such surfaces is an important element to preserve the smooth visual effect for the resulting triangulated surface. We reached the target by adopting[6]' s method named geometrically uniform sampling. The sampling order is to obey the general subdivision scheme, such as Loop scheme[8] in figure 4.



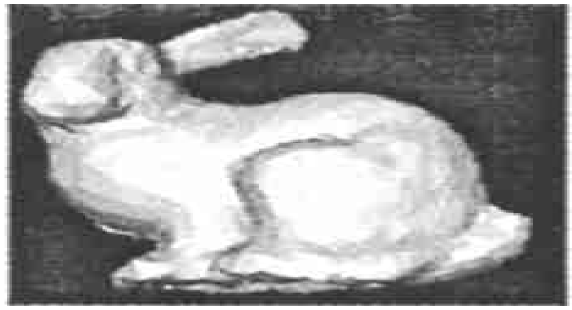
(a)



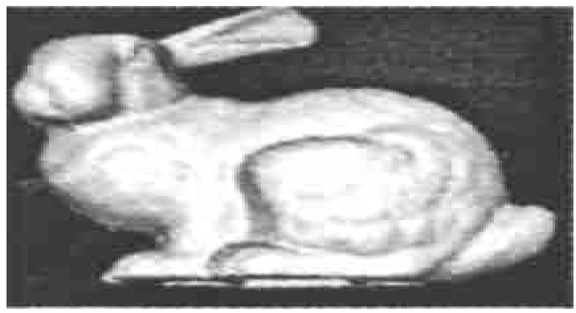
(b)



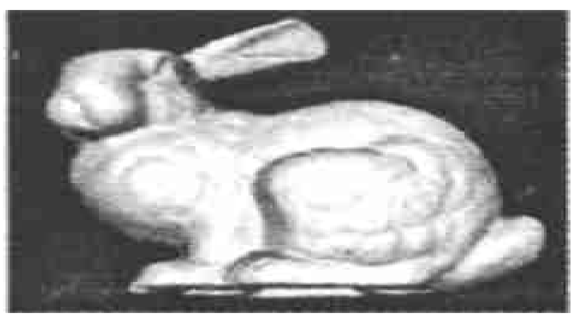
(c)



(d)



(e)



(f)

Fig. 4 Multiresolution representation of Stanford University' s bunny. (a) Object from laser scanner with 362272 points (b) Object from space-time analysis with 69451 faces. (c) Domains built by shape-preserving parameterization. (d) Geometrically sampling for 2 levels. (e) Firstly 3 levels geometrical sampling followed by 1 level parametrical sampling. (f) Firstly 3 levels geometrical sampling followed by 2 levels parametrical sampling.

5 Conclusion and Future Work

We have described the multiresolution surface construction method based on the parameterization named shape-preserving. During our parameterization, the fast Bi-CGSTAB is adopted as the solver for the linear equation system. Compared with the parameterization based on harmonic map, this parameterization effectiveness has been showed in the computing time and the sampling result from this parameter.

And yet, geometric features extraction and evaluation on such triangulated surface will be a fundamental problem. Recent work[13] has concerned with this problem. Combined with the dual local characters in time-frequency domain of wavelets, how to effectively use the wavelet coefficients to represent the geometric features will be an interesting context for our further work.

References:

[1] Gioia P. Reducing the number of wavelet coefficients by geometric partitioning[J]. Computational Geometry Theory and Applications, 1999, 14: 25-48.

[2] Eells J, Sampson L H. Harmonic mappings of Riemannian manifords[J]. Amer. J. Math, 1964, 86: 109-160.

[3] Duchamp T, Certain A, Derose T, Stuetzle W. Hierarchical computation of PL harmonic embeddings[C]. Tech. Rep, University of Washton, 1997.

[4] Barret O' Neill. Elementary differential geometry[C]. San Diego, USA: 2nd Academic Press, 1997.

[5] Stephane G. Mallat. A theory for multiresolution signal decomposition: The wavelet representation[J]. IEEE Transactions on Pattern Analysis and Machine Intelligence. 1989, 11(7): 674-693.

[6] Eck M, DeRose T, DUCHAMP T. Multi- resolution analysis of arbitrary meshes. ACM Computer Graphics[C]. SIGGRAPH' 95 Proceedings, Los Angeles, California, 1995, 173-182.

[7] Michael Lounsbery, Tony D. DeRose, Joe Warren. Multiresolution analysis for surfaces of arbitrary topological type[J]. ACM Transactions on Graphics, 1997, 16(1): 34-73.

[8] Loop, C T. Smooth subdivision surfaces based on triangles. Master' s thesis[A]. Dept. of Mathematics[C], Univ. of Utah. August, 1987.

[9] van H A DER VORS. BI-CGSTAB: A fast and smoothly converging variant of BI-CG for the solution of nonsymmetrical linear systems[J]. SIAM J. Sci. Stat. Comput, 1992, 13(2): 631-644.

[10] Aaron W. F. Lee. Peter Schroder, Lawrence Cowsar, David Dobkin. MAPS: multiresolution adaptive parameterization of surfaces [A]. In ACM Computer Graphics[C]. SIGGRAPH' 98 Proceedings, 1998, 19-24.

[11] Kai Hormann, Gunther Greiner. An efficient global parametrization Method[A]. Curves & Surfaces Fourth International Conference[C], France, 1999, 1-7.

[12] Allen Van Gelden. Approximate simulation of elastic memberances by triangulated spring meshes[J]. Journal of Graphics Tools, 1998, 3(2): 21-42.

[13] Christian Rossl, Leif Kobbelt, Hans-Peter Seidel, Extraction of feature lines on triangulated surfaces using morphological operators[A]. AAAI Spring Symposium, Smart Graphics[C]. Stanford University, 2000.

[14] Michael S. Floater. Parameterization and smooth approximation of surface triangulation[J]. Computer Aided Geometric Design, 1997, 14: 231-250.

[15] Kanai T, Suzuki H, Kimura F. Three-dimensional geometric metamorphosis based on harmonic maps[J]. The Visual Computer, 1998, 14(4): 166-176.

[16] Zhang D, Hebert M. Harmonic maps and their applications in surface matching[A]. IEEE Conference on Computer Vision and Pattern Recognition[C]. (CVPR' 99), 1999.

[17] Marc Levoy, Kari Pulli, et al. The digital michelangelo project: 3D scanning of large status[A]. ACM Computer Graphics[C]. SIGGRAPH 2000 proceedings, 2000.

基于小波理论的多重分辨率的曲面构造

郭　云[1], 山口 泰[1], 钟先信[2]

(1. 日本东京大学图形和计算机科学系, 日本 东京;

2. 中国重庆大学光电精密仪器系, 重庆　400044)

摘要: 高精度、非接触式的激光数字扫描仪的诞生, 使得人们能非常方便地获得高细节的物体形状。这种物体是以非结构化和浓密的网格存储在计算机中的。为了在科学、工程、艺术等领域有效地应用这种物体, 需要建立物体在计算机中有效表达形式。基于小波分析的多重分辨率的曲面构造能担任这种任务。细分连接(subdivision-connection)的小波函数的构造需要我们把曲面参数化到一个简单的复合形域中。在这篇文章中, 我们从微分几何的测地极映射观点出发, 提出用快速的形状保持的参数化方法有效地建立具有多重分辨率的物体曲面形状。同前面的方法相比, 我们的构造过程可以节约大量的计算时间且维持良好的物体曲面形状。

关　键　词: 多重分辨率曲面表达; 参数化; 测地极映射; 小波

中图分类号: TP391　　文献标识码: A

Resume: Guo Yun　was born in Jiangxi province, China in 1967. He is currently a PhD student, sponsored by Japanese Government Monbusho Scholarship and SaneYoshi Scholarship, in the Department of Graphics and Computer Sciences, the University of Yokyo. He received his BE and ME from Chongqing University of China in 1989 and 1992, respectively. In 1995, he entered the Doctor's course in the Department of Optical Precision Machine of Chongqing University. His research interests include the areas of computer graphics, computational geometry, wavelet analysis, and surface recognition. He has published several papers in the areas of computer graphics and modeling. He can be reached by e-mail: guo@graco.c.u-tokyo.ac.jp, or through postal address: Room D224, The University of Tokyo, Mitaka International Hall of Residence 6-22-20 Shinkawa, Mitaka-Shi, Tokyo 181-0004, Japan.

Yasushi Yamaguchi　was born in Japan in 1961. He is currently an associated professor in the Depamtent of Graphics and Computer Sciences, the University of Tokyo. His research interests lie in computer-aided design and geometric modeling, including parametric modeling, topology models for B-reps surface intersection and surface interrogation. He is a member of ACM SIGGRAPH, IEEE Computer Society, and SIAM. He received BE in precision machinery engineering and Dr. Eng. in information engineering from the University of Tokyo in 1983 and 1988, respectively. He was an assistant professor of Tokyo Denki University from 1989 to 1993. He has cooperated with Tokyo Research Lab, IBM, Stanford University in the areas of computer graphics and CAD geometric modeling. He has published many papers in the international professional journals such as *Computer Aided Geometric Design, IEEE Computer Society, ACM etc*. He can be reached through postal address: Department of Graphics and Computer Sciences/The University of Tokyo/3-8-1, Komaba, Meguro-ku/Tokyo 153-8902/Japan.